Bounds on the Size of Sound Monotone Switching Networks Accepting Permutation Sets of Directed Trees

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Abstract
Given a directed graph $G$ as input with labeled nodes $s$ and $t$, the ST-connectivity problem asks whether $s$ and $t$ are connected. The memory efficiency of an algorithm which solves this problem can be analyzed using sound monotone switching networks. This paper concerns bounds on the size of sound monotone switching networks which are restricted to the case that the input graph $G$ is isomorphic to a given graph $H$. Previously, tight bounds had been found in the cases that $H$ is a special kind of tree and that $H$ is a collection of disjoint paths from $s$ to $t$. This paper improves these results to find a nearly tight bound which applies to all directed trees. If we let $n$ be the number of vertices in the graph, $\ell$ be the length of the path from $s$ to $t$ in the tree, and $C_1$ and $C_2$ be variables which depend on the distances of the vertices from $s$ and $t$, then an upper bound on the size is on the order of $n \log \log \ell C_1 \log \ell$ and the lower bound is on the order of $C_2 \log \ell$. These two bounds are within $\log \log \ell$ times a constant factor in the exponent.

Summary
A graph is a collection of points with edges drawn between some of the nodes. Given a graph with one-way edges, we wish to compute whether there is a path from a starting point to an ending point. If we assume the computation is performed in a certain way, we can calculate roughly the minimal amount of computer memory needed. The problem under consideration is to estimate the amount of memory necessary for special families of graphs. Previously, reasonable estimates had only been found in specific cases. In this paper, we find estimates which apply to a significantly wider range of graphs.