Finite Range Couplings in a Tensor Renormalization Group Approach to 2D Classical Lattice Models

Luke Tang
Benjamin Franklin High School, New Orleans, LA, USA

under the direction of
Dr. Liang Fu
Massachusetts Institute of Technology

Abstract
The Ising model is a remarkable model which originated in physics and chemistry but has a wide range of applications in other fields. However, it makes a crucial approximation: all far-range interactions are ignored in the Ising model for the sake of simplicity, even though far-range interactions are almost always present. It is unknown to what degree the inclusion of these interactions will affect the dynamics of the Ising model. We generalize the tensor renormalization group (TRG) method of analyzing 2D classical lattices to account for next nearest neighbor (NNN) interactions in a square lattice. We find that NNN interactions do not in fact have a significant impact on the behavior of the square Ising lattice. This adapted TRG method more rigorously justifies the nearest neighbor approximation, however, and could help resolve the long-standing problem of geometrical frustration in 2D lattices.

Summary
The Ising model is a remarkably elegant and powerful model in physics and chemistry, and is capable of explaining phenomena as varied as neuron interactions and de facto racial segregation in cities. However, the Ising model makes a crucial approximation: all far-range interactions are ignored in the Ising model for the sake of simplicity, even though far-range interactions are almost always present. It is unknown to what degree the inclusion of these interactions will affect the dynamics of the Ising model. We generalize an existing method for analyzing the Ising model to account for farther range interactions, and find that these interactions do not in fact have a significant impact on the behavior of the model. This generalized method more rigorously justifies the exclusion of far-range interactions in the Ising model, however, and could also help resolve some long-standing problems in statistical physics.